Day 2
Lecture 1:
The SIR model

Short course on modelling infectious disease dynamics in R

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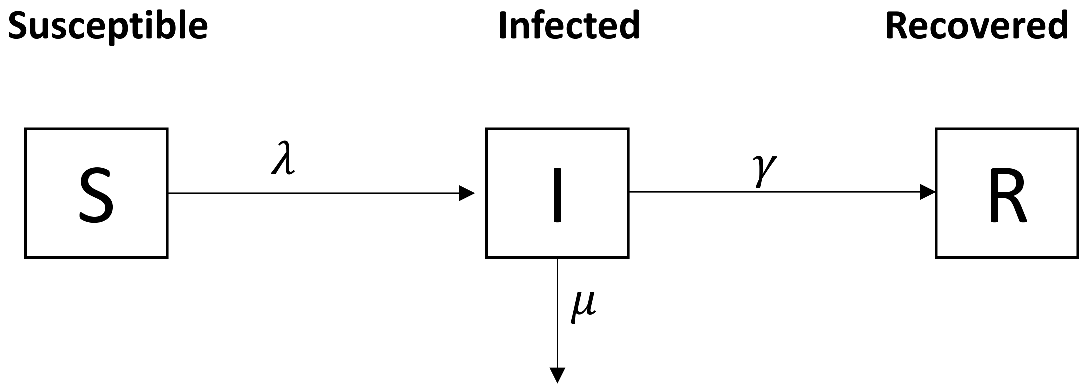
Aims of the session

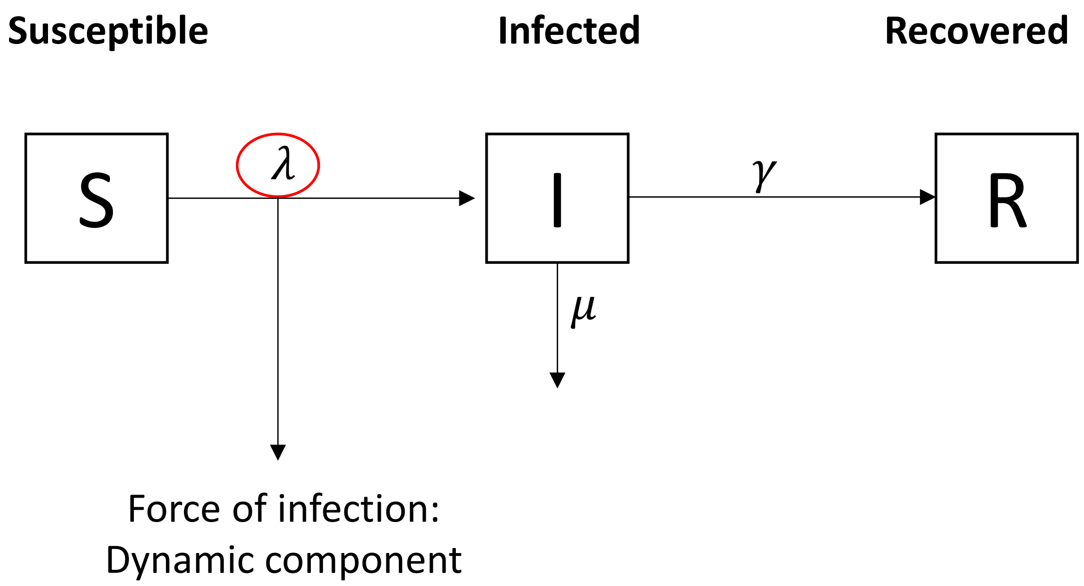
- Learn what is the process of transmission in the SIR model
- Understand the assumptions of the SIR model
- Describe the force of infection and its components

The SIR model



- We have explored linear transitions in a cohort model I -> R
- What does it take to introduce a process of transmission?

The Force of Infection (FOI)



- An increase in FOI should reflect an increase in the chance of any S meeting an infected person
- How do we do that?

The Force of Infection (FOI) deconstructed

$$\lambda =$$

The Force of Infection (FOI) deconstructed

Per contact probability of transmission

$$\lambda = p$$

The Force of Infection (FOI) deconstructed

Average contact rate per unit time

Per contact probability of transmission

$$\lambda = pc$$

The Force of Infection (FOI) deconstructed

Per contact probability of transmission

Average contact rate per unit time

Number infected

$$\lambda = pc\frac{I}{N}$$

I/N = Prevalence!!

Total population

The Force of Infection (FOI) deconstructed

Average contact rate per unit time

Number infected

Per contact probability of transmission

$$\lambda = pc\frac{I}{N}$$

Usually hard to estimate

Total population

The Force of Infection (FOI) deconstructed

Infection rate per unit time
$\beta = pc$

Number infected

$$\lambda = \frac{\beta I}{N}$$

Total population

The Force of Infection (FOI) deconstructed

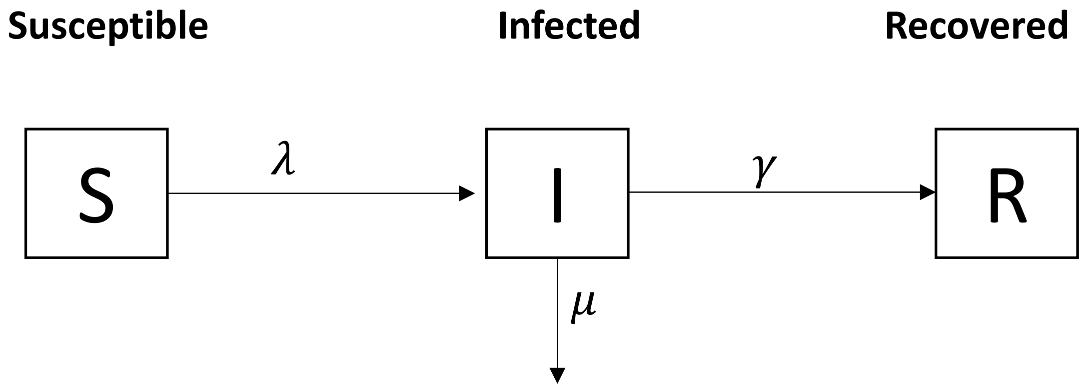
Infection rate is a constant

I is a state variable

$$\lambda = \frac{\beta I}{N}$$

- A small ***I*** means a small **λ**
- FOI changes over time
- We allow **λ** to be a function of other model variables

Transitions in the SIR model



Linear transition I to R:

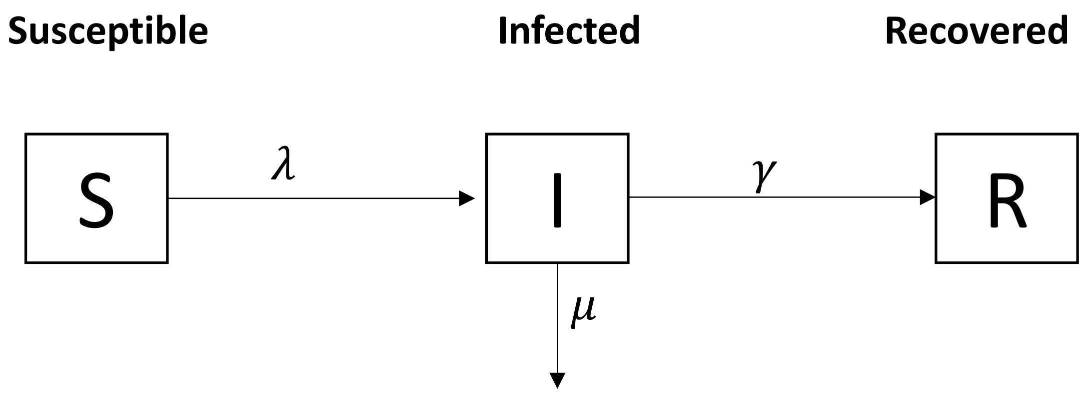
$$I\gamma$$

Non-linear transition S to I:

$$S\lambda = S\beta\frac{I}{N}$$

This is the engine of the transmission model!!

Differential equations for the SIR



$$\frac{dS}{dt} = -S\lambda(t)$$

$$\frac{dI}{dt} = S\lambda(t) - I\mu - I\gamma$$

$$\frac{dR}{dt} = I\gamma$$

Same as..

$$\frac{dS}{dt} = -S\beta\frac{I}{N}$$

$$\frac{dI}{dt} = S\beta\frac{I}{N} - I\mu - I\gamma$$

$$\frac{dR}{dt} = I\gamma$$

Lessons from the SIR model

1) If $\lambda = pc\frac{I}{N}$, then the same infection in a population with a larger *c* will result in a higher force of infection

2) Similarly, biological factors of the pathogen affecting *p* mean that different infections will have different FOI in the same population

Assumptions in the SIR model

1) Transition $S\lambda$ assumes homogeneity in risk of infection. (think why!)

2) If $\lambda = pc\frac{I}{N}$, means that the fraction of infected contacts a person can meet every day is equal to the prevalence (well mixed population).

   a) We should expect that a fraction infected (ill people) will be isolated or in less contact

   b) This is the assumption of homogeneity in contact

3) It is possible that *p* can be estimated at a much granular level (think why!)

What we should know by now

- What is an SIR model
- What is the force of infection
- What are the components of the force of infection
- What are the main assumptions behind the SIR model